Chien-Shiung Wu's Contribution to the Standard Model

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- Fundamental quantum condition:

$$\boxed{pq - qp = \frac{h}{2\pi i}}$$

where the Planck constant is $h = 6.626 \times 10^{-34}$ J s. It is inscribed on the gravestone of Max Born (1882–1970). The picture was taken by Professor Jeremy Bernstein's colleague at Göttingen.

- Let $q = x$, $p = \dfrac{\hbar}{i}\dfrac{d}{dx}$, where $\hbar = h/(2\pi)$ is the reduced Planck constant,

$$(pq - qp)\psi(x) = \frac{\hbar}{i}\frac{d}{dx}[x\psi(x)] - x\frac{\hbar}{i}\frac{d}{dx}\psi(x) = \frac{\hbar}{i}\psi(x) + x\frac{\hbar}{i}\frac{d}{dx}\psi(x) - x\frac{\hbar}{i}\frac{d}{dx}\psi(x),$$

the last two terms cancel each other, and we have $(pq - qp)\psi(x) = \dfrac{\hbar}{i}\psi(x)$. A wavefunction $\psi(x)$ is introduced in this representation.

- Schrödinger equation:

$$\boxed{H\Psi = i\hbar\frac{\partial}{\partial t}\Psi}$$

where H is the Hamiltonian operator. In classical mechanics, H is the sum of kinetic energy and potential energy V. For a particle moving in one dimension, the kinetic energy is $p^2/(2m)$ where p is the momentum and m the mass. With the above differential operator for p, the Schrödinger equation is written as

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}.$$

- Einstein's energy-momentum relation:

$$\boxed{E^2 = (pc)^2 + (mc^2)^2}$$

In 1905, Einstein actually wrote

$$m = \frac{E}{c^2}$$

- Four-dimensional spacetime:
$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$$
The spacetime interval ds^2 is invariant. The great mathematician Emmy Noether said Einstein's theory of relativity should be called **theory of invariance**; see the last footnote of her 1918 paper `https://arxiv.org/abs/physics/0503066`. Energy together with 3 components of momentum form the energy-momentum four vector, $p^\mu = (E/c, p_x, p_y, p_z)$. The energy-momentum relation can be written as
$$p^\mu p_\mu - (mc)^2 = 0$$

- Dirac equation (free):
$$\boxed{i\hbar\gamma^\mu \partial_\mu \psi - mc\psi = 0}$$
Essentially Dirac tried to "factor" the energy-momentum relation $p^\mu p_\mu - (mc)^2 = 0$, but by doing so he needed to introduce the 4×4 γ matrices. The result was $p^\mu p_\mu - (mc)^2 = (\gamma^\alpha p_\alpha + mc)(\gamma^\beta p_\beta - mc) = 0$. With the substitution $p_\mu \to i\hbar\partial_\mu$, we obtained the Dirac equation.

- Lagrangian mechanics:
$$S = \int_{t_i}^{t_f} L\, dt, \quad \delta S = 0,$$
where the Lagrangian L is the kinetic energy *minus* potential energy, and S is called the action. This formulation is the same as the familiar
$$\mathbf{F} = m\mathbf{a}$$
where $\mathbf{F}$ is the force and $\mathbf{a}$ the acceleration.

- Dirac Lagrangian (density) with $c = \hbar = 1$:
$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

- Quantum electrodynamics:
$$\boxed{S_{\text{QED}} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi \right]}$$
where $D_\mu = \partial_\mu + ieA_\mu$ is the gauge covariant derivative, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field tensor. All of chemistry and biology can be reduced, *in principle*, to the laws of quantum electrodynamics.

- Parity violation in weak interaction: In 1956, Tsung-Dao Lee and Chen-Ning Yang proposed that the weak interaction does not preserve parity. It was experimentally verified by Chien-Shiung Wu. The pictures show Professor Wu at her lab and with family in front of her apartment on Claremont Avenue near Columbia University. Image credits: Columbia University and the Washington Post.


- Electroweak unification: After the Wu experiment confirming parity violation in the weak interaction, a search began for a new theory. In 1967, Steven Weinberg published a paper titled "A Model of Leptons" (*Physical Review Letters*, **19**, 1264–1266). He posited that an $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge group could unify electromagnetism and the weak interactions. His model is (with a minor typo fixed)

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu\right)^2 - \frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2 - \bar{R}\gamma^\mu\left(\partial_\mu - ig' B_\mu\right)R - \bar{L}\gamma^\mu\left(\partial_\mu - ig\vec{t}\cdot\vec{A}_\mu - i\frac{1}{2}g' B_\mu\right)L - \frac{1}{2}|\partial_\mu\varphi - ig\vec{A}_\mu\cdot\vec{t}\varphi + i\frac{1}{2}g' B_\mu\varphi|^2 - G_e\left(\bar{L}\varphi R + \bar{R}\varphi^\dagger L\right) - M_1^2\varphi^\dagger\varphi + h\left(\varphi^\dagger\varphi\right)^2,$$

where the left-handed doublet is

$$L \equiv \left[\frac{1}{2}(1+\gamma_5)\right]\begin{pmatrix}\nu_e \\ e\end{pmatrix}$$

and the right-handed singlet is

$$R \equiv \left[\frac{1}{2}(1-\gamma_5)\right]e.$$

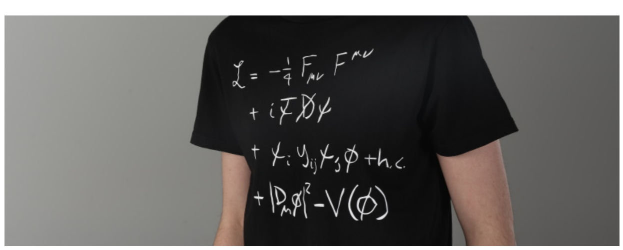
The spin-zero doublet (Higgs) is

$$\varphi = \begin{pmatrix}\varphi^0 \\ \varphi^-\end{pmatrix}$$

By 1976, this paper has become the most cited high-energy physics paper on record, a position it held for more than 30 years. `https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.19.1264`

- Quantum chromodynamics: In 1973, David Gross, Frank Wilczek, and David Politzer developed a theory analogous to the quantum electrodynamics for the strong nuclear force, using the non-abelian Yang-Mills gauge theory with symmetry group $\mathrm{SU}(3)_C$. The Lagrangian looks similar to that of QED, but the field tensor is $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\mathfrak{c}_{abc}A^b_\mu A^c_\nu$ where g is the gauge coupling constant and $\mathfrak{c}_{abc}$ are the structure constants of the group. In the picture taken at the University of Illinois in 1966 you can find the young Frank Wilczek and Gordon Crandall.

- Standard model: A T-shirt visitors can buy at CERN.



- Core Theory: Frank Wilczek has been advocating the promotion from the Standard Model to the Core Theory. In Sean Carroll's words, "The laws underlying the physics of everyday life are completely understood." From `https://www.preposterousuniverse.com/blog/2013/01/04/the-world-of-everyday-experience-in-one-equation/`, the action is written as

$$W = \overbrace{\int_{k<\Lambda} [Dg][DA][D\psi][D\Phi] \exp}^{\text{quantum mechanics}} \Bigg\{ i \overbrace{\int d^4x \sqrt{-g}}^{\text{spacetime}} \Bigg[\overbrace{\frac{m_p^2}{2} R}^{\text{gravity}}$$

$$\underbrace{-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{other forces}} + \underbrace{i\bar{\psi}^i \gamma^\mu D_\mu \psi^i + \left(\bar{\psi}^i_L V_{ij} \Phi \psi^j_R + \text{h.c.} \right)}_{\text{matter}} \underbrace{- |D_\mu \Phi|^2 - V(\Phi)}_{\text{Higgs}} \Bigg] \Bigg\}$$

- Fermion mass term: If we decompose a Dirac field into left and right handed fields, $\psi = \psi_L + \psi_R$, the Dirac Lagrangian becomes

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R - m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

where the notation $a\!\!\!/ \equiv \gamma^\mu a_\mu$ was invented by Richard Feynman. The mass term connects left to right and right to left. In the Standard Model, the weak interactions violate parity, so such a term is forbidden by gauge symmetry.

The Higgs field solves the problem. A fermion term (Yukawa coupling) can be formed as

$$\mathcal{L}_{\text{fermion}} = -\bar{\psi}_L Y \Phi \psi_R$$

After symmetry breaking, $\Phi \to \bar{\Phi} + H$, and the Higgs-fermion interaction becomes

$$\bar{\psi}_L Y \Phi \psi_R \to \bar{\psi}_L Y \bar{\Phi} \psi_R + \bar{\psi}_L Y H \psi_R$$

In the Dirac equation, the mass m is *a priori* arbitrary, but in the Standard Model, the mass is identified as $Y\bar{\Phi}$ and is linked to the Higgs vacuum expectation value $\bar{\Phi} = 1/\sqrt{\sqrt{2}G_F} = 1/\sqrt{\sqrt{2} \times 1.1663788 \times 10^{-5}\,\text{GeV}^{-2}} \approx 246.22$ GeV, where G_F is the Fermi coupling constant.[1]

Summary

Within 100 years after the invention of quantum mechanics, physicists have constructed a Core Theory that embodies the Standard Model of particle physics and Einstein's general relativity. The standard model possesses internal symmetry of $\text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$; Emmy Noether's 1918 theorem connects symmetries and conservation laws and has profoundly influenced modern physics. The Core Theory has been tested in regimes that included all of the energy scales relevant to the physics of everyday life (biology, chemistry, technology, etc.).[2] The laws of physics underlying the phenomena of everyday life are completely known. Professor C. S. Wu's discovery of parity violation in weak interaction was pivotal in the development of the Core Theory, as the Yukawa term in the Lagrangian density of the theory explicitly shows a left and right disparity.[3]

[1] In Weinberg's 1967 paper, λ is the symbol for $\bar{\Phi}$ that we adapted here, and $\lambda^2 = M_1^2/(2h)$, where M_1^2 and h are coefficients in the Higgs potential $-M_1^2 \varphi^\dagger \varphi + h\left(\varphi^\dagger \varphi\right)^2$ in the original Lagrangian; he used G_e for electron as our Y in this note. In his notation, the electron mass is expressed as λG_e, and the Higgs boson mass should be $2M_1$, which has been measured to be 125.20 GeV. All numerical values are from the Particle Data Group `https://pdg.lbl.gov`.

[2] Sean M. Carroll, The Quantum Field Theory on Which the Everyday World Supervenes, `https://arxiv.org/pdf/2101.07884`.

[3] In addition to her seminal role in demystifying the weak interaction, Professor Wu was a pioneer in testing "quantum entanglement," described by Schrödinger as *the* characteristic trait of quantum mechanics. This phenomenon is what makes a quantum computer powerful. To learn more, see Chien-Shiung Wu's Trailblazing Experiments in Particle Physics by Chon-Fai Kam, Cheng-Ning Zhang, and Da Hsuan Feng in the 2024 December issue of *Physics Today*, `https://doi.org/10.1063/pt.oufp.zwkj`. See also `https://www.cswuforum.org`.